# A PHYSICAL INTERPRETATION OF THE POISSON WAVELET TRANSFORM OF POTENTIAL FIELDS

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Hornby *et al.* (1999) derive a Cartesian coordinate wavelet specialised for potential fields from the horizontal gradient of the Green's function for the vertical acceleration due to a point mass. In this framework, the wavelet transform of a potential field at some height $z_0$ is given by the simple expression:

$$\overrightarrow{W}\left[f_0\right](s,\overrightarrow{x}) = (z/z_0)\,\overrightarrow{\nabla} f_z\left(\overrightarrow{x}\right)$$

where $f_0$ is the original potential field on a plane, the wavelet scale $s = z/z_0$, and $\overrightarrow{\nabla}$ denotes the vectorial 2D horizontal gradient. It has been shown in Appendix B of Hornby *et al.* (1999) that the inverse of this wavelet transform (the IWT) is given by:

$$f_0\left(\overrightarrow{x}\right) = 4\int_0^\infty \frac{ds}{s}\int_{\Re^2}\left(\overrightarrow{\psi_s}\left(\overrightarrow{u}-\overrightarrow{x}\right),\overrightarrow{W}\left[f_0\right](s,\overrightarrow{u})\right)_{\Re^2} d\overrightarrow{u}$$

Here, $\overrightarrow{\psi_s}$ is the wavelet basis function proportional to the field due to the horizontal gradient of a point source at depth (i.e. proportional to the field due to a mass dipole at depth), and $(\sim,\sim)_{\Re^2}$ denotes a 2D inner product.

Now, because the function $\overrightarrow{\psi_s}$ is both the analysing *and* the synthesising wavelet for this problem, there is an interesting physical interpretation of the function $\overrightarrow{W}$. *The wavelet transform itself is proportional to a mass dipole source distribution — one that exactly generates the observed field* $f_0$.

Even though the form of the components of the dipole strength $\overrightarrow{W}$ is suggestive of probability amplitudes, it would be more correct to interpret the maxima of the horizontal gradients of the field at a given height as the points of greatest edge density in a particular IWT source model — which provides a physical basis for traditional techniques such as those of Cordell and Grauch. The divergence of the dipole distribution (a vector field) is, by Gauss' theorem, a valid density distribution. The PDE satisfied by this density distribution can be deduced. From this PDE we can in turn deduce a variational term that leads to this equation, and hence infer the prior hypothesis being expressed. While the solution so obtained is not itself of great practical interest, it is interesting to note the implication that a limiting solution to a Tikhonov stabilisation can be obtained directly from a wavelet transform.

The application of this technique to magnetic field strengths is most simply accomplished via the pseudo-gravity transform of the observed field. Many examples will be displayed at the meeting.


1. Hornby, P., F. Boschetti, and F.G. Horowitz, "Analysis of Potential Field Data in the Wavelet Domain", Geophysical Journal International, v. 137, pp. 175-196, 1999.