TRUTH IN ERRORS: A SIMPLE UNIFYING RELATIONSHIP BETWEEN LEAST SQUARES AND MAXIMUM LIKELIHOOD ERRORS IN ISOCHRON ANALYSIS

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The overwhelming number of radiometric ages have been and will continue to be determined by fitting a straight line to a set of experimentally measured points on the x-y plane (with possible correlations between x and y errors). Obviously then, in the search for "true" ages, it is important that we have a full understanding of this line-fitting process. It has long been recognized that the Least Squares Estimation (LSE) method of fitting the best straight line to data points having normally distributed errors yields the identical result for the slope and intercept as does the method of Maximum Likelihood Estimation (MLE). However, we have found that, contrary to previous understanding, these two techniques also give identical results (algebraically and therefore numerically) for the standard errors in slope and intercept. The MLE error expressions are seen to be special solutions of the York (1969) LSE error equations, despite the apparently very different analytical approaches to error estimation adopted in the standard LSE and MLE approaches. As a result, we can now see for the first time the beautifully simple conceptual relationship between the classical LSE approach and the more recent MLE method of estimating errors. This allows us to present the solution to the line-fitting problem in the most compact form yet achieved for the set of four equations giving the slope, intercept and newly unified standard errors. Using a variety of data sets, we have also compared these analytical error estimates with the results of Monte Carlo trials with ten million iterations. The resulting close agreement of the analytical and Monte Carlo results indicates that the unified analytical error estimates are extremely accurate.